Quantum mechanics

Time Independent Perturbation Theory

- The stationary perturbation theory , is concerned with finding the changes in the discrete energy levels and eigen function of a system, when a small disturbance is applied
- The Hamiltonian 'H' in the schrodinger wave equation can be written as the sum of two parts
- $H = H_o + H'$
- Where H_o is the unperturbed Hamiltonian
- H' is the perturbation term

- We expand the perturbed eigen function and eigen value as power series in H'
- The perturbed wave function and energy level and written.
- $\psi_n = \psi_n^\circ + \lambda\psi'_n + \lambda^2 \psi^2_n + \ldots\ldots..$
- $E_n = E_n^\circ + \lambda E'_n + \lambda^2 E_n^{2} + \ldots\ldots..$
- The above equations are substituted into the wave equation to give
- $(H_\circ + \lambda H')\,(\psi_n^\circ + \lambda \psi'_{n+} \lambda^2 \psi^2_n) = (E_n = E_n^\circ + \lambda E'_{n+} \lambda^2 E_n^{2} + \ldots\ldots..)\,(\psi_n^\circ + \lambda\psi + \ldots\ldots.)$
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- Then, we can equate the coefficients of equal power of λ on both sides to obtain a series of equation
- $H^\circ \psi_n^\circ = E_n^\circ \psi_n^\circ$
- $H' \psi_n^\circ + H^\circ \psi_n' = E' \psi_n^\circ + E_n^\circ \psi_n'$
- $H' \psi_n' + H^\circ \psi_n^2 = E'' \psi_n^\circ + E'_n \psi_n + E_n^\circ \psi_n^2$

First-order correction to the energy

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  Multiplying the first order equation from the left by $<\psi_n^o|$, we get

- $$< \psi_n^o|H'|\psi_n^o> + < \psi_n^o|H'|\psi_n'> = E_n' <\psi_n^o/\psi_n^o> +$$
- $E_n^o < \psi_n^o/\psi_n'>$

  From the above equation we get,

  $$E_{n'} = < \psi_n^o|H'|\psi_n^o>$$
- The first order correction to the energy is thus the average value of the perturbation over the corresponding unperturbed states of the system.

Second order correction to the energy.

- The second order correction to the energy from equation is
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  $H'\psi_n{}' + H^{\circ}\psi_n{}^2 = E_n{}^2\psi_n{}^{\circ} + E_n{}'\,\psi_n{}' + E_n{}^{\circ}\,\psi_n{}^2$

  multiplying the above equation from left by
- $<\psi_n{}^{\circ}|$ weget,
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  $<\psi_m{}^{\circ}|H'|\,\psi_n{}'> = E_n{}^2 <\psi_n{}^{\circ}\,|\,\psi_n{}^{\circ}> + E_n{}' <\psi_n{}^{\circ}\,|\,\psi_n{}'>$

$$E_n{}^2 = <\psi_n{}^{\circ}|H'|\,\psi_n{}'>$$

- substituting the value of ψ_n' ,we get,

- $$E_n^2 = \frac{<\psi_m^o|H'|\psi_n^o> \psi_n^o|H'|\psi_m^o>}{E_n^o - E_m^o}$$

$$E_n^2 = \frac{|<\psi_m^o|H'|\psi_n^o>|^2}{E_n^o - E_n^o}$$

- The second order correction in energy to level n due to levels for which $E_n^o > E_n^o$ is postive whereas that due to levels for which iwhich $E_n^o < E_n^o$ is negative.