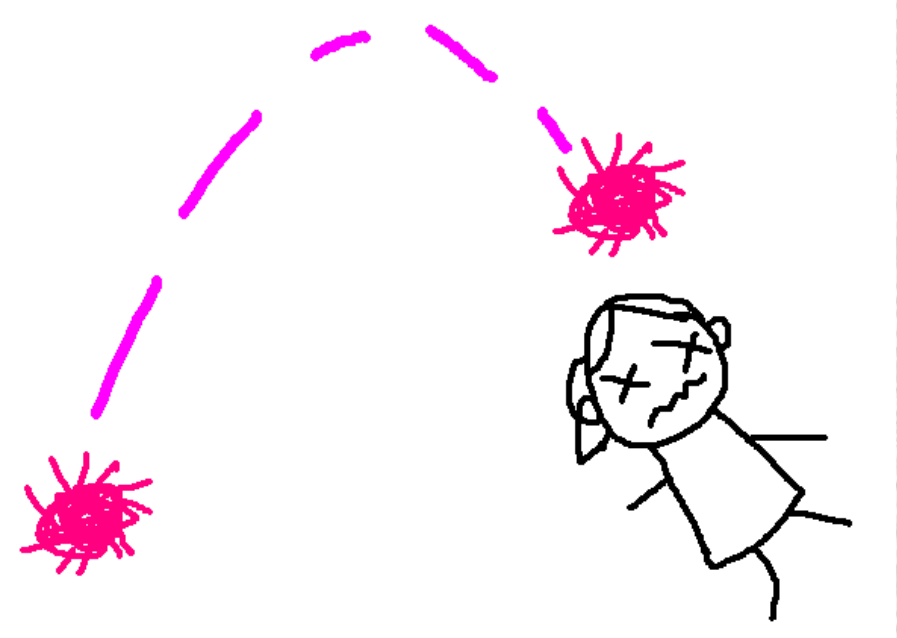
St. John's College, Palayamkottai

Department of Mathematics

Dynamics –Dr. S. Shyamala Malini

What is projectile?

Any object which projected by some means and continues to move due to its own inertia (mass).



Projectile Motion:

Motion through the air without a propulsion

Angle of Projection

The angle of projection is the angle that the direction in which the particle is initially projected makes with the horizontal plane through the point of projection.

Velcoity of Projection:

Velocity of Projection is the velocity with which the particle is projected.

Trajectory

The trajectory is the path which the particles describes.

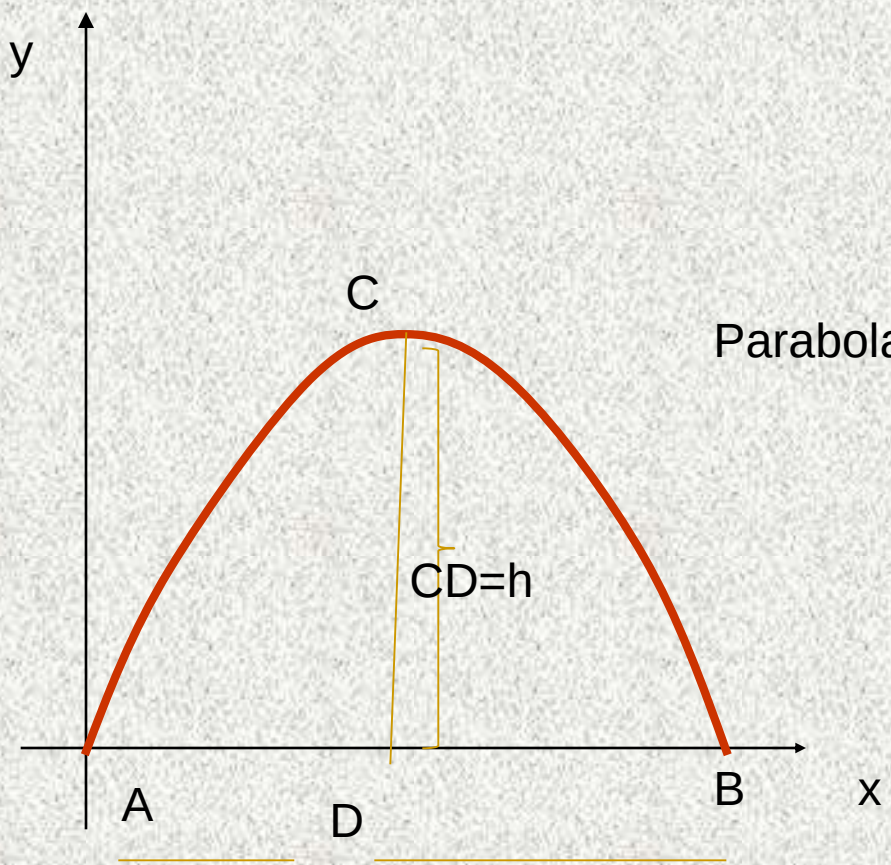
Range:

Range on a plane through the point of projection is the distance between the point of projection and the point where the trajectory meets that flight.

Example

Time of flight:

The time of flight is the interval of time that ellapses from the instant of projection till the instant when the particle again meets the horizontal plane through the point of projection.



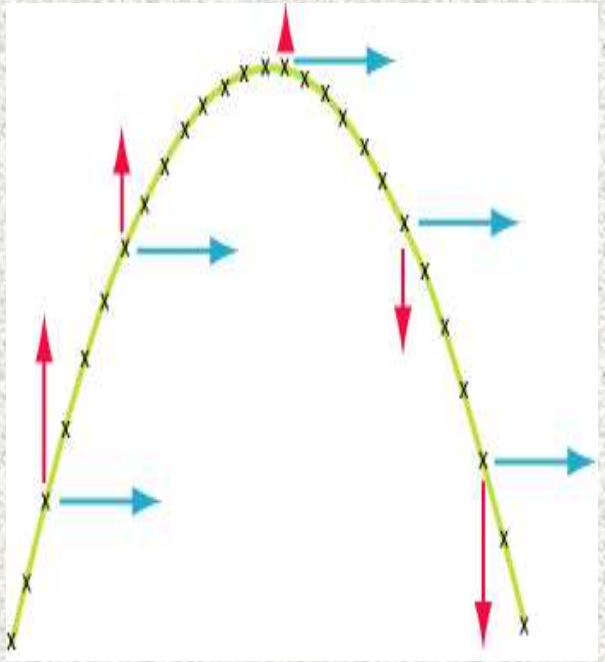
Parabola, open *down*

Range=
AB

Projectiles move in 2-dimensions

Since a projectile moves in 2-dimensions, it therefore has 2 components just like a resultant vector.

- **Horizontal and Vertical**

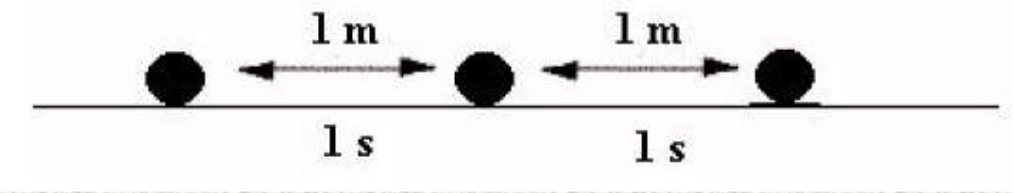


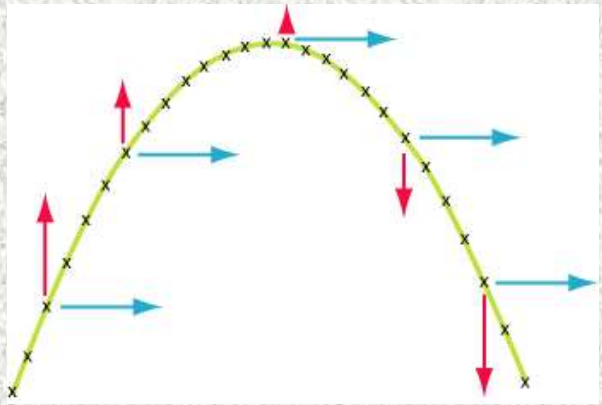
Equation of motion through the laws of constant acceleration. Hence, these equations are used to derive the components like displacement(s), velocity (initial and final), time(t) and acceleration(a).. The three equations are,

- $v = u + at$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2}at^2$

Horizontal "Velocity" Component

- **NEVER** changes, covers equal displacements in equal time periods. This means the initial horizontal velocity equals the final horizontal velocity



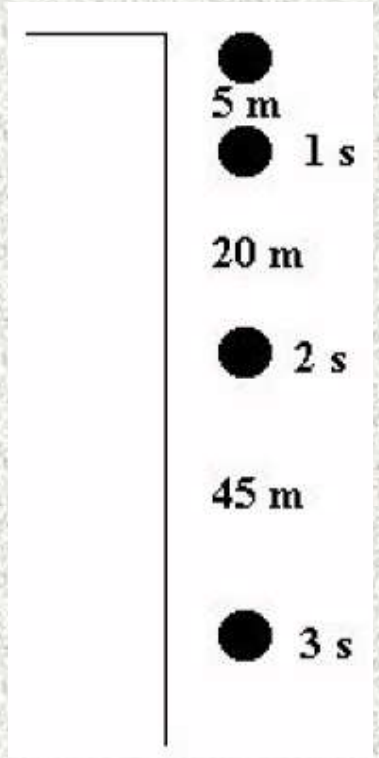


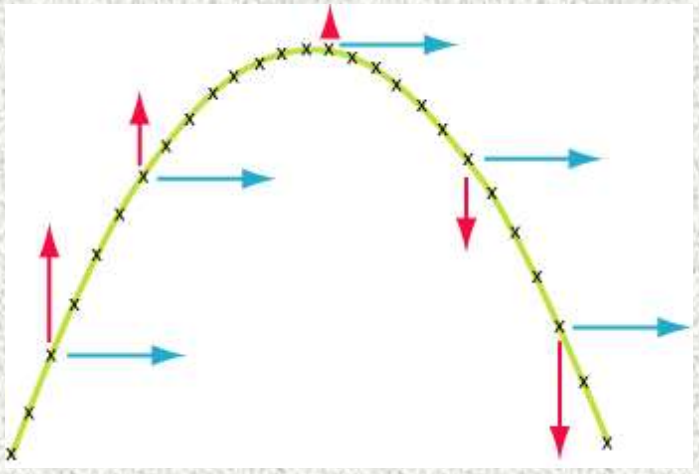
In other words, the horizontal velocity is CONSTANT. BUT WHY?

Gravity DOES NOT work horizontally to increase or decrease the velocity.

Vertical "Velocity" Component

- Changes (due to gravity), does **NOT** cover equal displacements in equal time periods.



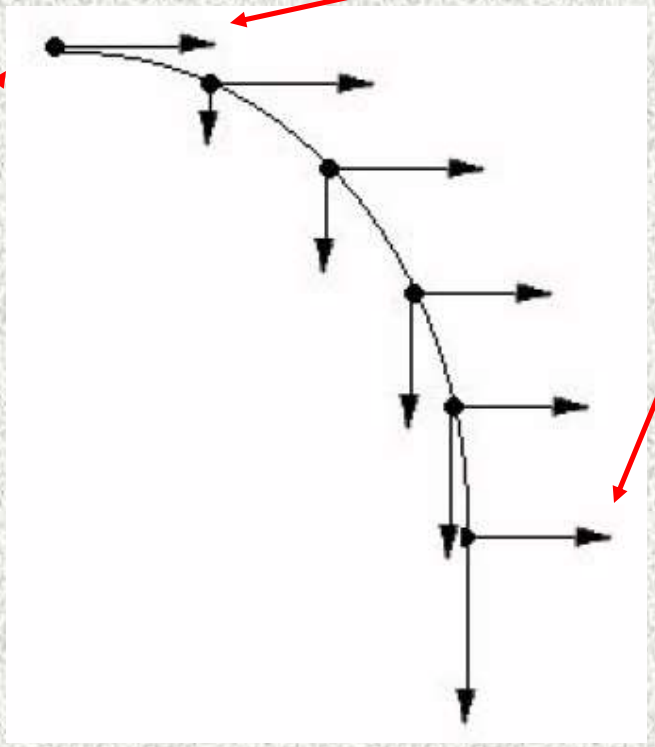


Both the MAGNITUDE and DIRECTION change. As the projectile moves up the MAGNITUDE DECREASES and its direction is UPWARD. As it moves down the MAGNITUDE INCREASES and the direction is DOWNWARD.

Horizontally Launched Projectiles

Projectiles which have NO upward trajectory and NO initial VERTICAL velocity.

$$v_{ox} = v_x = \text{constant}$$



$$v_{oy} = 0\, m/s$$

Horizontally Launched Projectiles

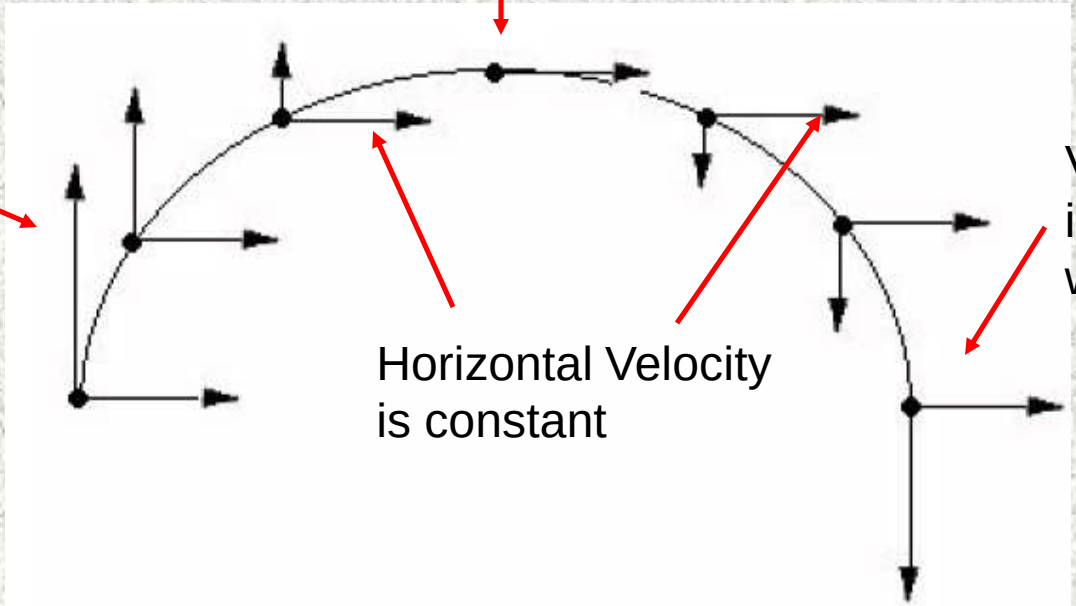
To analyze a projectile in 2 dimension we need 2 equations. One for the “x” direction and one for the “y” direction. We use Kinematic Law

$$S = ut + \frac{1}{2}at^2$$

$x = ut$	$y = \frac{1}{2}gt^2$
Remember, the velocity is CONSTANT horizontally, so that means the acceleration is ZERO!	Remember that since the projectile is launched horizontally, the INITIAL VERTICAL VELOCITY is equal to ZERO.

Vertically Launched Projectiles

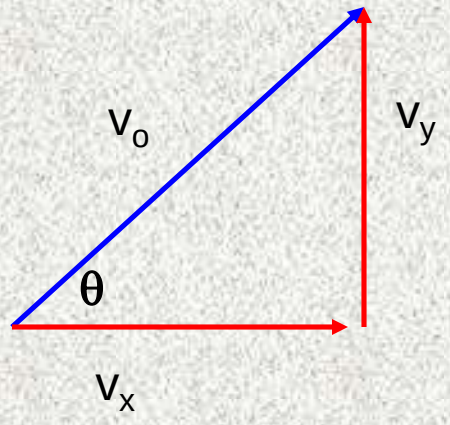
NO Vertical Velocity at the top of the trajectory.



Component	Magnitude	Direction
Horizontal	Constant	Constant
Vertical	Decreases up, 0 @ top, Increases down	Changes

Vertically Launched Projectiles

Since the projectile was launched at a angle, the velocity MUST be broken into components!!!

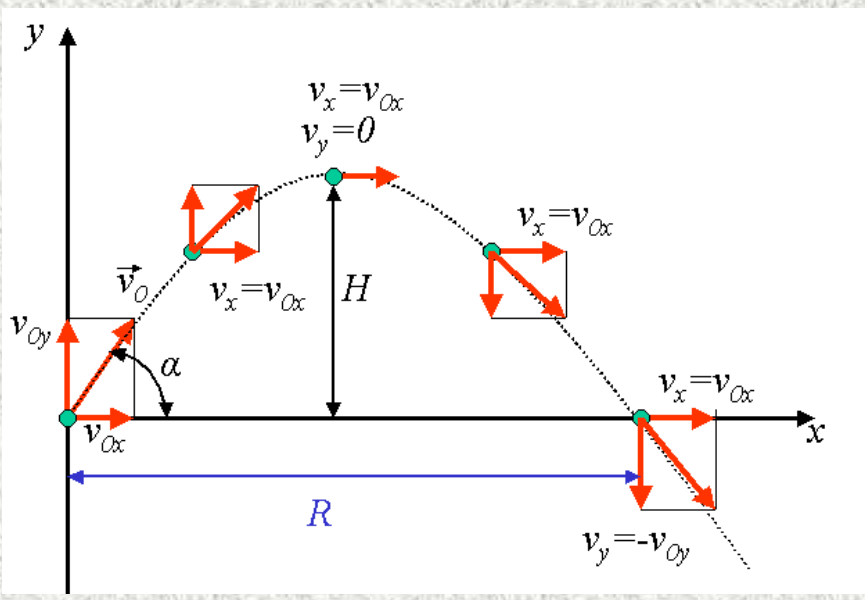


$$v_x = u\cos\alpha$$

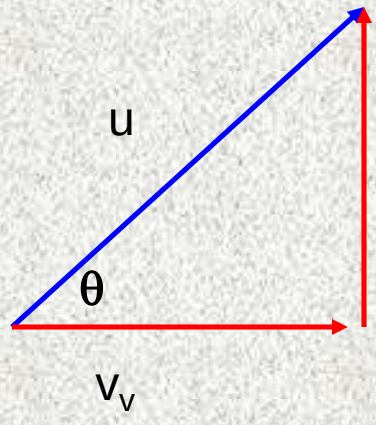
$$v_y = u\sin\alpha$$

Vertically Launched Projectiles

There are several things you must consider when doing these types of projectiles besides using components. If it begins and ends at ground level, the “y” displacement is ZERO: y = 0



Vertically Launched Projectiles



$$x = v_x t \qquad y = v_y t - \frac{1}{2} g t^2$$

$$v_x = u \cos \alpha$$

$$v_y = u \sin \alpha$$

Path of a Projectile is a Parabola

$$x = u\cos\alpha t \quad \text{---------(1)}$$

$$y = u\sin\alpha t - \frac{1}{2}gt^2$$

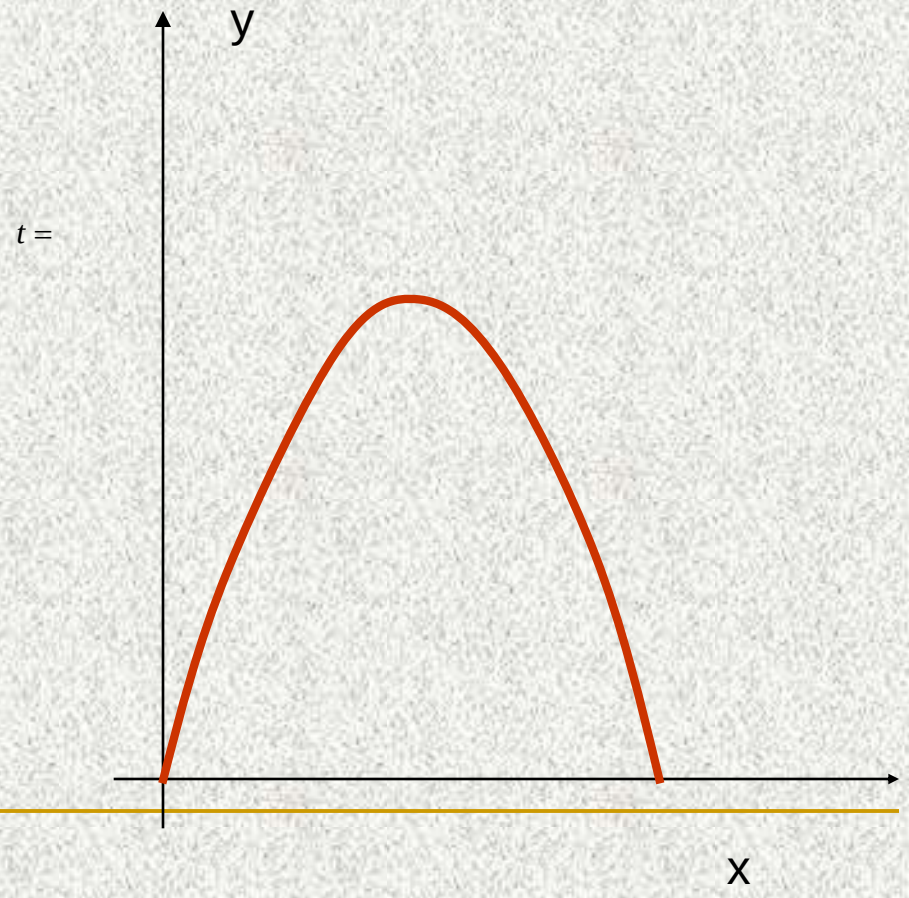
Eliminate time, t

$$t = \frac{x}{u\cos\alpha}$$

$$y = \frac{ux\sin\alpha}{u\cos\alpha} - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$y = x\tan\alpha - \frac{g}{2u^2\cos^2\alpha}x^2$$

Parabola, open *down*

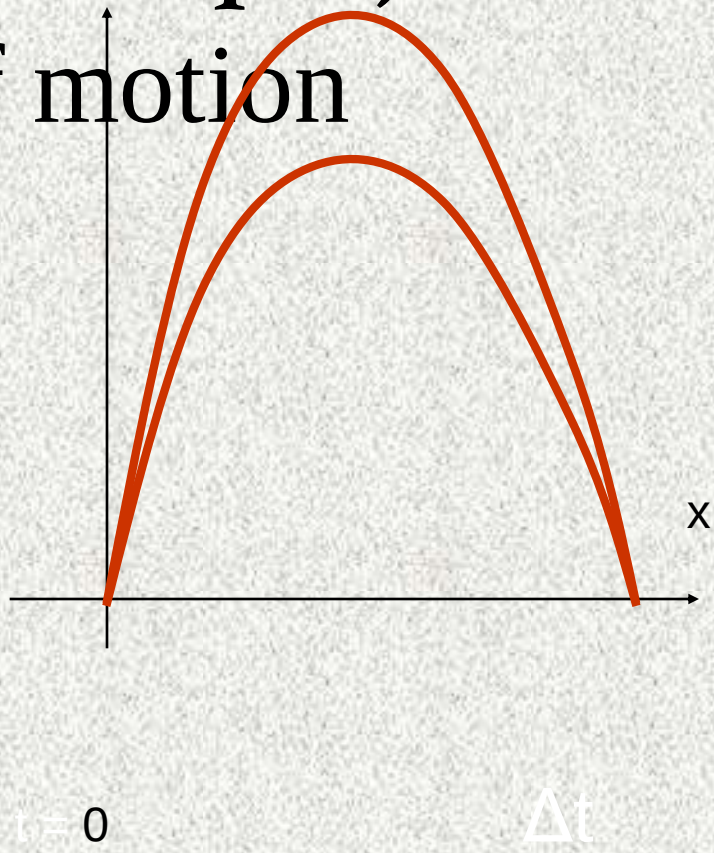


Greatest height attained by a projectile

Using the equation of motion equation 2,

$$0 = u^2 \sin^2 \alpha - 2hg$$

$$h = \frac{u^2 \sin^2 \alpha}{2g}$$




BAR & GRILL

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Time taken to reach the greatest height

$$u \sin \alpha - gt = 0$$

$$T = \frac{u \sin \alpha}{g}$$

Time of flight

$$2u \sin \alpha - gt = 0$$

$$t = \frac{2u \sin \alpha}{g}$$

Horizontal range on an inclined plane

$$Horizontal velocity = u\cos\alpha t \text{ ------(1)}$$

$$subt = \frac{2u\sin\alpha}{g} ineqn(1) weget$$

$$R = u\cos\alpha\frac{2u\sin\alpha}{g}$$

$$R = \frac{2u^2\sin\alpha\cos\alpha}{g} = \frac{u^2\sin 2\alpha}{g}$$

Thank you